X-PAD Ultimate FAQ Series

State Plane Coordinates at Ground, method 3:
LCC 2P to LCC 1P with Scale and Offset

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Thesis

Perhaps the best way to generate a Modified State Plane Coordinate system at ground at a specified Base Point for a Lambert Conformal projection is to convert the 2-Parallel LCC Grid Projection to a 1-Parallel LCC Ground Projection, then apply the correct combined scale factors to bring the projection to ground at the Base Point.

This method has the advantage of working in all software packages with a minimum number of system coefficients:

Origin Lat, Origin Lon, False Northing, False Easting and a single Scale factor

While this FAQ includes screenshots for X-PAD, it will work for all other field and desktop software packages.

Setup a Modified SPC Projection at Ground

Consider two points, West and East. The western point will be the Base Point for our projection. The eastern point will be used as a checkpoint.

Western Point: The Base Point

```
Lat Lon Ellipsoid:          37 07 48.88043 N 113 30 35.44965 W NAD8      (D M S.s)     (1)
                            37.13024456388 N 113.50984712500 W          (D.d)
                            2786.820 Ft orthometric; 2706.895 (825.062 m)ellip          (2)

UT South SPC (sFeet):       10,017,594.0413 SFt N 1,054,514.4670 SFt E UTS NAD83 (3)
                            10017594.0413 N       1054514.4670 E
```

We don't want to confuse future surveyors with coordinates that are nearly identical to SPC coordinates, lets choose coordinates that do not match the SPC coordinates by only using the 6 most significant digits of the SPC coordinates:

```
Truncated Local Coord:      017,594.0413 SFt N   054,514.4670 SFt E                     (3)
```

However, because the leading digit is 0 in this case, our new coordinate system would have a negative Northing if we moved south more than 3.3 miles and a negative Easting if we move west more than 10.3 miles; so, let's just make the leading digits 5 and 3. This should clearly distinguish the Northing values (which will start with 5) and Easting values (which will start with 3):

```
Fixed Local Coord:          517,594.0413 SFt N   354,514.4670 SFt E                     (4)
```

These will be the desired Northing and Easting coordinates for the Base Point in our new system. They should be close enough 'looking' to the SPC coordinates to remind us-and-future surveyors where they were derived from.

Eastern Point:

This eastern point will be used as a checkpoint. Note that the projected coordinates of the eastern point will change as a result of bringing it to ground.

```
Lat Lon Ellipsoid:            37 7 48.57847 N 113 30 04.12079 W NAD83         (5)
                              2804.71 Ft orthometric 2724.840 ellipsoid       (6)

UT South SPC (sFeet):  10,017,509.1018 SFt N   1,057,050.3304 SFt E           (7)
```

Inversing from West to East:

```
       Bearing: S88°04'53.685527"E   Grid: 2537.2855ft                        (8)
```


We expect that the resulting ground system will have exactly the same bearing and that the distance will be lengthened to match a ground distance.

Addition Notes and Expectations

NGS NCAT Tool Check

The NGS NCAT tool (https://geodesy.noaa.gov/NCAT/) is useful for checking scale factors. For the Western base point:

Converted Coordinate

Reference Frame:NAD83(2011)

Lat-Lon-Height		SPC		UTM/USNG		XYZ (m)	
Latitude	N37° 07' 48.88043' N370748.88043 37.1302445639	Zone	UT S-4303	Zone	12	X	-2,031,208.418
Longitude	E246° 29' 24.55035' W1133035.44965 -113.5098471250	Northing	3,053,368.770 (m) 10,017,594.041 (usft) 10,017,614.076 (ift)	Northing (m)	4,112,269.745	Y	-4,669,264.867
Ellipsoid Height (usft)	2706.894	Easting	321,416.652 (m) 1,054,514.467 (usft) 1,054,516.576 (ift)	Easting (m)	277,045.630	Z	3,829,425.087
		Convergence (dms)	-01 13 53.07	Convergence (dms)	-01 30 56.31		
		Scale factor	1.00001594	Scale factor	1.00021239		
		Combined factor	0.99988648	Combined factor	1.00008290		
				USNG	12STG7704612270		

From NCAT:

```
       GridSF:               1.00001594                                        (11)
       CcombinedSF:          0.99988648                                        (12)
```

thus

```
       ElevationSF = CSF/GSF 0.99985254           (computed)                   (13)
       1/CSF:                1.00011353                                        (14)
```

Using the NGS CSF we expect the ground distance to be:

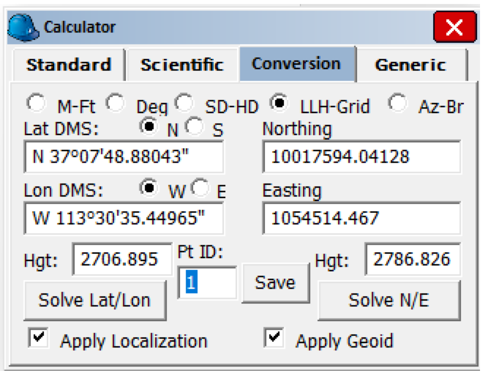
```
       Ground: 2,537.5736 ft = (Grid) 2537.2855 ft * (1/CSF) 1.00011353        (15)
```

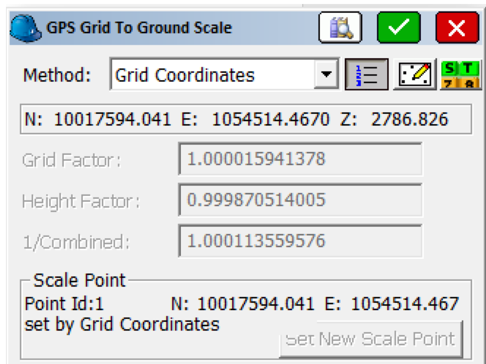
Checking this:

Using a TM projection: 2537.574

Using Carlson SurvCE: 2537.574

Here are some screenshots from Carlson showing computed factors:





From Carlson:

```
GSF:                  1.000015941378                                           (16)
ESF:                  0.999870514005               ( NGS -> 0.99985254 )       (17)
CSF:                  0.9998865453319      1/CSF:        1.000113559575        (18)
```

Compute an elevation scale factor (ellipsoidal reduction) for the Base Point

If you don't want to use NCAT to find the Elevation Scale Factor, the correct equation for the earth radius at a latitude:

$$r = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\phi}$$

where, for GRS 1980 (and NAD83);

a = semi-major axis (radius of equator) = 6,378,137.000 m

e^2 = eccentricity squared = 0.00669438002290.

ϕ = geodetic latitude. (21)

and the elevation scale factor is:

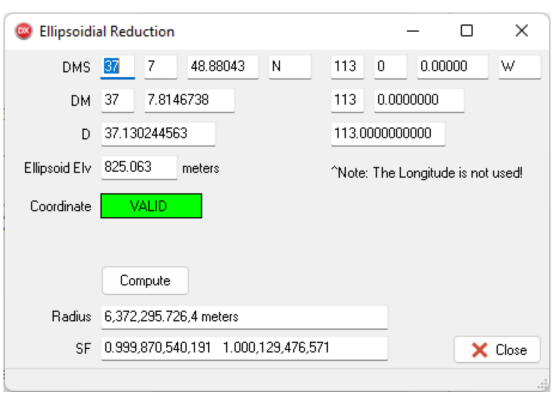
```
ESF     = Rg / (Rg + H)                                                          (21.1)
```

where Rg is the radius and H is the height above the ellipsoid.

I have included code in Appendix A to compute radius and the elevation scale factor, however, you could also use my 'Ellipsoid Reduction' too:

https://iggps.com/out/utilities/EllipsoidialReduction/index.htm

Enter the Latitude and Ellipsoid Elevation of the Base point:



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```
Radius at 37 07 48.88043 N = 6,372,295.726,4 meters                                 (22)
ESF:            0.999,870,540,191      0.999870540191 ( NGS -> 0.99985254 )         (23)
1/ESF:          1.000,129,476,571      1.000129476571                               (24)
```

The NGS uses a different method to compute the Elevation Scale Factor. The slight difference between the NGS value, my value and SurvCE's value is inconsequential for the relatively short distance in this example.

Note: this is NOT a Combined Scale Factor for the State Plane projection. It is the ellipsoid reduction factor and will need to be combined with the grid scale factor to get a combined scale factor.

Get the coefficients for the single parallel LCC

Use the NOAA Manual NOS NGS5, State Plane Coordinate System of 1983 by James E Stern, 1989.

On page 113 the Coefficients for a single parallel LCC are listed for Utah South NAD83 zone:

```
B0      =       37.7840696241(D.d)      37 47 02.65064676 (D M S.s)                 (31)
K0      =       0.999,951,297,078       0.999951297078                              (32)
```

Alternatively, you can compute the center latitude and scale factor, tested code is included in Appendix B of this document.

Get the Point Scale Factor for the Base Point

The Grid Scale Factor at a point can be directly computed.

Lambert Conformal Conic (LCC) projection

The grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos\varphi_C}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_C}} \times \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C} - \ln\frac{1+\sin\varphi}{1-\sin\varphi} + e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi} - \ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right)\right]\right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if $k_0 = 1$)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

f = geometric flattening of the reference ellipsoid

(33)

Appendix C contains a tested code example that will compute the Point Grid Scale Factor. Solving for the Point Grid Scale Factor for the Base Point:

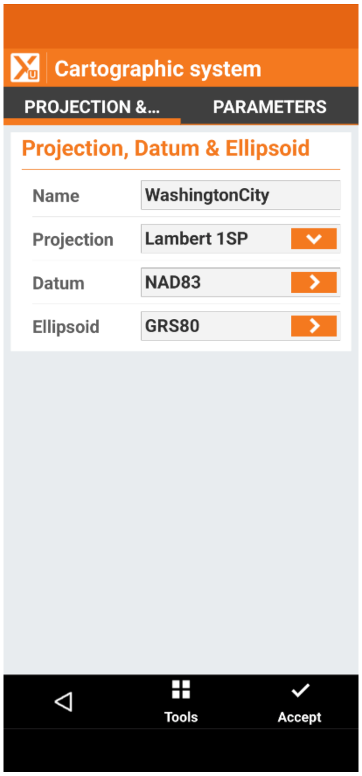
```
K       =       1.000,015,941,378           1.000015941378                          (34)
```

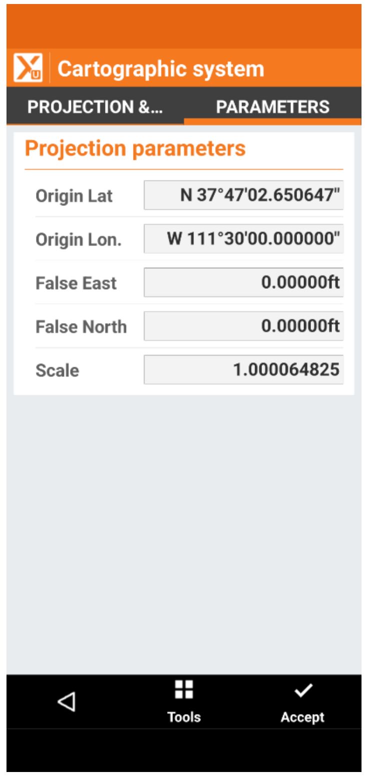
Compute a Ground Scale Scale Factor

Compute a new scale factor for the single parallel LCC that adjusts for the Point Scale Factor and the Ellipsoid Reduction Factor:

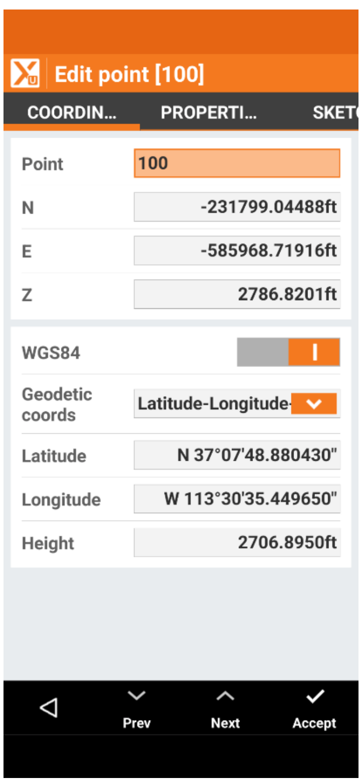
```
CSF     =       ( K0 / PointScaleFactor ) * ( 1 / ElevationScaleFactor )
        =       K0 / ( PointScaleFactor * ElevationScaleFactor )
        =       0.999951297078 / ( 1.000015941378 * 0.999870540191 )
        =       (32)           / ( (34)           * (23)           )
        =       1.000064824932                                                      (35)
```

Enter these values into a new Lambert 1SP projection with 0.0 false northing and eastings:





Return to the Points listing for the western Base Point:

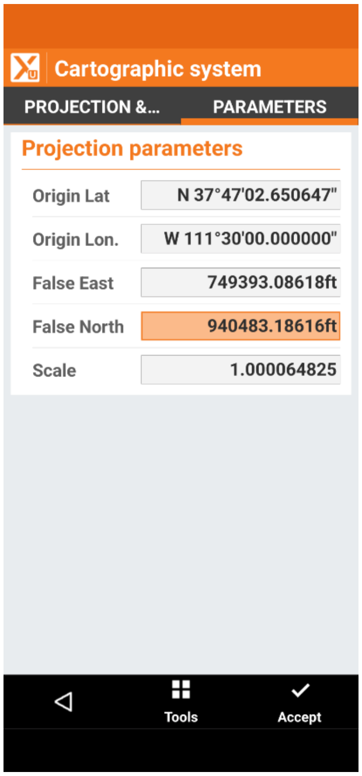


Note the projected Northing and Easting values.

Compute the required **False Northing** and **False Easting**:

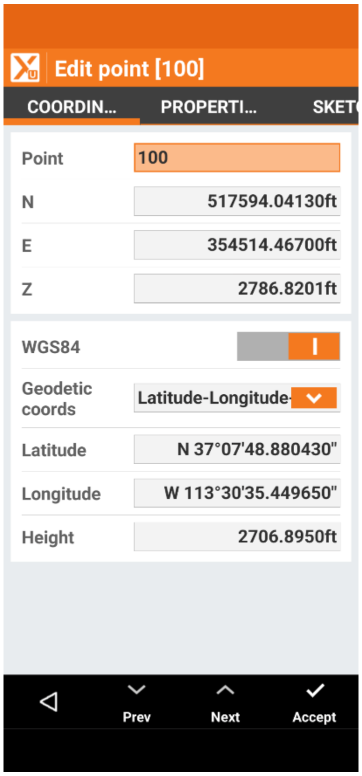
```
False Northing = 517594.0413 – (-231799.04488)
               = 749393.08618
False Easting  = 354514.4670 – (-585968.71916)
               = 940483.18616
```

Return to the Coordinate system, Projection entry screen and enter the **False Northing** and **False Easting** values:

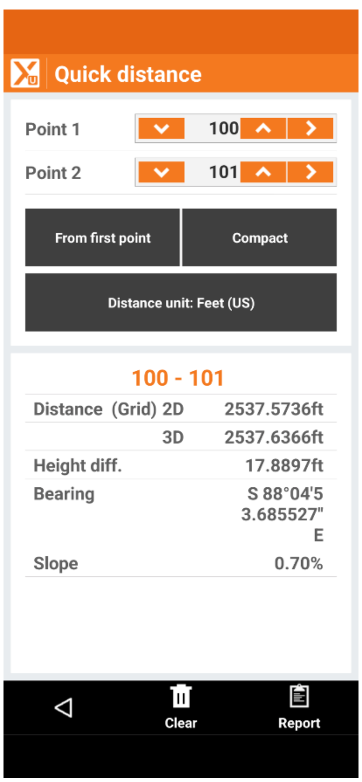


Be careful in X-PAD, the **Easting** is on top of the **Northing**.

Accept the changes, the return to the point listing and make sure the point has the correct projected values:



Finally Inverse (**Quick Distance**) between the western and eastern points:



Note that the distance and bearing match our expectations exactly:

```
Ground: 2,537.5736 ft = (Grid) 2537.2855 ft * (1/CSF) 1.00011353          (15)
```

Since we know the projection is correct, return to the Coordinate system. Cartographic system and save the definition with a name so that it can easily be reused with other jobs. Use the **Tools** button, then click **Save as predefined**.

Note

It is possible to directly compute the False Northing and Easting knowing the scale factors and desired values however it is far easier to just do the two subtractions as shown above. This method results in exact results for the Base Point, regardless of small changes in scale factors.

Appendix A: Compute Elevation Scale Factor

$$r = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\phi}$$ where, for GRS 1980 (and NAD83);

a = semi-major axis (radius of equator) = 6,378,137.000 m

e^2 = eccentricity squared = 0.00669438002290.

ϕ = geodetic latitude.

```
function EllipsoidReduction( Lat: extended;   // Latitude (degrees)
                              ElevM: extended;  // ellipsoid elevation (meters)
                             var Rg: extended   // Radius at lat
                             ): extended;       // returns the elevation scale factor
var
  st: extended;
  a,f,e2: extended;
begin
  // GRS80
  a := 6378137.0;
  f := 1.0 / 298.257222101; // GRS80
  e2 := (f * (2.0 - f));
  // Sin of Theta             radians
  st := sin( DegToRad(Lat) );
  // Geometric Mean Radius of Curvature
  Rg := a * sqrt( 1.0 - e2 ) / (1.0 - e2 * st * st );
  // Ellipsoidial Reduction
  result := Rg / (Rg + ElevM );
end;
```

Appendix B: 2 to 1 Parallel, Point Grid Scale Factor

Convert a two parallel LCC to the equivalent single parallel LCC.

From ‘Ground Truth for the Future, Low Distortion Projections and the State Plane Coordinate System of 2022’, Michael L. Dennis, August 2019, pg 31 and ‘State Plane Coordinate System of 1983’, NOS NGS 5, James E Stern, January 1990:

$$\varphi_C = \sin^{-1}\left[\frac{2\ln\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}}{\ln\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}-\ln\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}+e\left(\ln\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}-\ln\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)}\right]$$

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_C}\sqrt{\frac{1-e^2\sin^2\varphi_C}{1-e^2\sin^2\varphi_N}} \times \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_N}{1-\sin\varphi_N} - \ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C} + e\left(\ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C} - \ln\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]\right\}$$

```
procedure TwoPtoOneP( LatNd, LatSd: extended;  // N & S parallels in degrees
                  var LatCd, K0: extended );   // return single central parallel in deg,
                                               // equivalent scale factor at central parallel
var
  LatN, LatS: extended;  // N & S parallels in radians
  SinN, SinS, CosN, CosS, SinC, CosC: extended; // temps
  a, f, e, ee, q1, q2, w1, w2, sin0: extended;  // temps
  // resulting values
  LatC: extended; // lattitude of single parallel radiuns, degrees

begin
  // Convert to radians
  LatN := DegToRad( LatNd );
  LatS := DegToRad( LatSd );

  // Precompute Sin, Cos
  SinN := sin( LatN );
  SinS := sin( LatS );
  CosN := cos( LatN );
  CosS := cos( LatS );

  // Earth radius, flattening ratio for GRS80
  a    := 6378137.0;
  f    := 1.0/298.257222101;
  // precompute  e^2 and e
  ee   := 2*f - f*f;
  e    := sqrt( ee );

  // Compute one-parallel central parallel from two-parallel
  // Isometric latitude of standard parallels. From MDennis, 09/15/2022
  // example: Using Lat1 = 44.00 and Lat2 = 42.12, for the GRS 80 ellipsoid
  // Lat0  =  43.062436753128252947050822037 5 = 43°03'44.772311262"N
  q1 := ( ln((1 + SinN)/(1 - SinN)) - e * ln((1 + e*SinN)/(1 - e*SinN)) ) / 2;
  q2 := ( ln((1 + SinS)/(1 - SinS)) - e * ln((1 + e*SinS)/(1 - e*SinS)) ) / 2;

  // Sine and latitude of central parallel.
  w1 := sqrt(1 - ee*power(SinN, 2));
  w2 := sqrt(1 - ee*power(SinS, 2));
  sin0 := (ln((w1*CosS) / (w2*CosN))) / (q1 - q2);

  LatC := arcsin(sin0); // Solve for the single parallel latitude
  LatCd := RadToDeg( LatC ); // Convert to degrees for return value

  // Compute central parallel Scale Factor
  // for the example, k0 should = 0.999865901971223740
  // precompute the sin and cos of the single lat
  SinC := sin( LatC );
  CosC := cos( LatC );

  // compute in pieces, first term
  w1 := (CosN/CosC) * sqrt( (1-ee*power(SinC,2)) / (1-ee*power(SinN,2)) );

  // everything inside the square brackets (see page 32 of quaff)
  w2 :=       ln((1+SinN)/(1-SinN)) - ln( (1+SinC)/(1-SinC) ) +
        e * ( ln((1+e*SinC)/(1-e*SinC)) - ln((1+e*SinN)/(1-e*SinN)) );

  // combine for the single parallel scale factor
  k0 := w1 * exp( (SinC/2) * w2 );

end;
```

Appendix C: Grid Scale Factor for Single Parallel LCC

Knowing the center latitude and scale factor of a single parallel Lambert Conformal Conic projection, calculate the grid scale factor of a point with a given latitude:

Lambert Conformal Conic (LCC) projection

The grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos\varphi_C}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_C}} \times \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C} - \ln\frac{1+\sin\varphi}{1-\sin\varphi} + e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi} - \ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right)\right]\right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if $k_0 = 1$)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

f = geometric flattening of the reference ellipsoid

```
procedure PointKC( LatCd, K0, LatPd: extended;  // lat of central parallel, projection grid
                                         // scale factor for central parallel, lat of point
                   var k: extended );    // result GSF, grid scale factor at point
var
  LatC, LatP: extended; // radians
  a, f, ee, e: extended;
  tmp, C, SR: extended;
  sinc, sinp: extended;  // sin(LatC), sin(Lat)
begin
  // convert inputs from degrees to radians
  LatC := DegToRad( LatCd );   // lat of standard parallel
  LatP := DegToRad( LatPd );   // lat of the point

  // precompute
  sinc := sin(LatC);  // sin of the central parallel latitude
  sinp := sin(LatP);  // sin of the point latitude

  // Earth radius, flattening ratio for GRS80
  a    := 6378137.0;
  f    := 1.0/298.257222101;
  ee   := 2*f - f*f;
  e    := sqrt( ee );

  // Stem, 1990, pp. 26-29
  // Everything in the exp{ ... } part
  tmp := (sinc/2) *
           ( ln((1+sinc)/(1-sinc)) - ln((1+sinp)/(1-sinp)) +
             e*( ln( (1+e*sinp)/(1-e*sinp) ) - ln( (1+e*sinc)/(1-e*sinc) ) )
           );
  // ratio of cos's
  C   := cos(LatC)/cos(LatP);
  // the sqrt portion
  SR  := sqrt( (1 - ee*power(sinp,2)) / (1 - ee*power(sinc,2)) );

  // Compute the point scale factor result
  k := K0 * C * SR * exp( tmp );

end;
```